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Generation and Transfer of Quantum Entangled State via Spin-Parity Measurements

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Abstract We propose two schemes for entanglement generation and quantum state transfer via a double-quantum-dot system. Our schemes only need spin-parity measurements and single-qubit measurements combined with additional qubits. Discussions about the feasibility of the current scheme show that they would be realized within the current experimental technology.

Keywords Parity measurement · Entanglement generation · Quantum state transfer · Double-quantum-dot molecule

1 Introduction

Quantum entanglement is a unique phenomena only allowed in quantum mechanics. Quantum entangled state is considered as a universal resource for quantum information processing (QIP) because of its quantum state superposition principle and quantum nonlocality. Higher-dimensional quantum entangled state becomes more important for QIP, since it can violate local realism more intensively than two-dimensional entangled state, can store more information and can improve the noise limit of tolerant-error in quantum cryptography. Recently, multipartite cluster state attracts much attention and becomes very usefulness for quantum computation, especially for the one-way computer. Schemes for generating cluster states

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have been researched in different quantum systems, such as cavity QED [1–4], trapped ions systems [5], quantum dot systems [6, 7] and superconducting quantum systems [8–10]. On the other hand, quantum state remote transfer is regarded as a building block in quantum communication and is widely researched [11–14] after Bennet et al. proposed the first proposal of quantum teleportation [15]. So the generation and transfer of entangled states are both the basis tool for QIP.

Solid-state system is one of the best promising candidates for realizing quantum processors. Solid-state system based on electronic spin is considered as the best "hardware" for fabricating quantum computer. Thus semiconductor quantum dot system is investigated widely [16–19]. The decoherence time of electron-spin state is longer than that of electron orbit due to intrinsic character of solid state. In particular, qubit, encoded on the electron-spin singlet state and triplet state of double-quantum-dot molecules (DQDM), has longer decoherence time than others spin states [20, 21]. Furthermore, it can be protected from low-frequency noise and can suppress the dominant source of decoherence from hyperfine interaction [22, 23]. Quantum measurement will become very important as quantum information develops. Very recently, spin-parity meter has attracted much attention and has been widely discussed, which is nondestructive measurement and can avoid the routine Bell state measurement [24, 25]. Due to these advantages, we proposed two schemes based on spin singlet state and triplet state of double-quantum-dot molecules, and spin-parity measurements. One is the generation of multipartite entangled states, the other is the transfer of quantum state.

2 The Generation of Entangled States

In order to generate a N-partite cluster state, we initially prepare N DQDMs being in the state $|\phi\rangle_{1,2,\ldots,N} = \frac{1}{2^{N/2}} \bigotimes_{i=1}^{N} (|S\rangle_i + |T_0\rangle_i)$, and $N-1$ additional DQDMs being in the state $|T_0\rangle^{\otimes(N-1)}$, where $|S\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$ and $|T_0\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$. Then we perform a Hadamard operation on DQDM 1, which can be expressed as

$$|T_0\rangle_1 \to \frac{1}{\sqrt{2}}(|T_0\rangle_1 - |S\rangle_1) = |\uparrow\downarrow\rangle_1 \tag{1a}$$

$$|S\rangle_1 \to \frac{1}{\sqrt{2}}(|T_0\rangle_1 + |S\rangle_1) = |\downarrow\uparrow\rangle_1 \tag{1b}$$

Next we perform a spin-parity measurement on DQDM 1 and additional DQDM 1, which is shown in Fig. 1 (the right electron of DQDM 1 and the left electron of additional DQDM 1 are used as "witness" states). If the two spin states are identical, the spin-parity measurement result is even, otherwise, it is odd. If the result is odd ($P_{s,1} = 0$), the state of DQDM 1 and additional DQDM 1 becomes

$$|\varphi\rangle_{1a_1} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\downarrow\rangle_{1a_1} + |\downarrow\uparrow\downarrow\uparrow\rangle_{1a_1}) \tag{2}$$

If the result is even ($P_{s,1} = 1$), the state of DQDM 1 and additional DQDM 1 will be

$$|\psi\rangle_{1a_1} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\downarrow\uparrow\rangle_{1a_1} + |\downarrow\uparrow\uparrow\downarrow\rangle_{1a_1}) \tag{3}$$

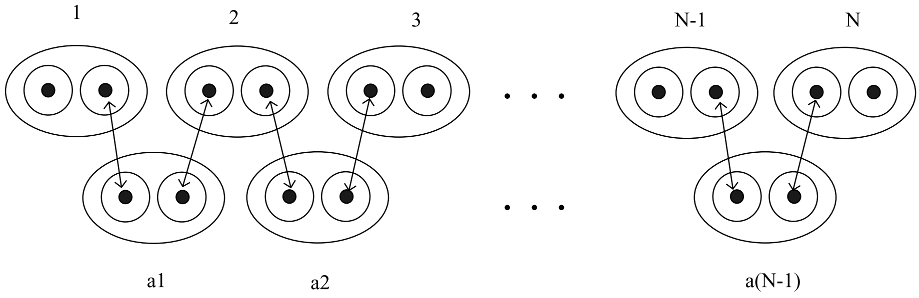

Fig. 1 Each DQDM concludes two quantum dots with only one electron. *Arrowhead* denotes spin-parity measurement

Then we perform a Hadamard operation on DQDM 1 and additional DQDM 1, respectively, which leads (2) and (3) to

$$|\varphi\rangle_{1a_1} = \frac{1}{\sqrt{2}}(|T_0T_0\rangle_{1a_1} + |SS\rangle_{1a_1}) \tag{4a}$$

$$|\psi\rangle_{1a_1} = \frac{1}{\sqrt{2}}(|T_0S\rangle_{1a_1} + |ST_0\rangle_{1a_1}) \tag{4b}$$

Then we perform a Hadamard operation on DQDM 2 as that in (1) and a spin-parity measurement on DQDM 2 and additional DQDM 1 (the left electron of DQDM 2 and the right electron of additional DQDM 1 are used as "witness" states). In the case of $P_{s,1} = 0$ and $P_{s,2} = 0$, the state of DQDMs 1, 2 and additional DQDM 1 will be

$$|\phi\rangle^a_{12a_1} = \frac{1}{2}[(|S\rangle_1 + |T_0\rangle_1)|\downarrow\uparrow\rangle_{a_1}|\downarrow\uparrow\rangle_2 + (|T_0\rangle_1 - |S\rangle_1)|\uparrow\downarrow\rangle_{a_1}|\uparrow\downarrow\rangle_2] \tag{5}$$

In the case of $P_{s,1} = 0$ and $P_{s,2} = 1$,

$$|\phi\rangle^b_{12a_1} = \frac{1}{2}[(|T_0\rangle_1 - |S\rangle_1)|\uparrow\downarrow\rangle_{a_1}|\downarrow\uparrow\rangle_2 + (|T_0\rangle_1 + |S\rangle_1)|\downarrow\uparrow\rangle_{a_1}|\uparrow\downarrow\rangle_2] \tag{6}$$

In the case of $P_{s,1} = 1$ and $P_{s,2} = 0$,

$$|\phi\rangle^c_{12a_1} = \frac{1}{2}[(|S\rangle_1 + |T_0\rangle_1)|\downarrow\uparrow\rangle_{a_1}|\downarrow\uparrow\rangle_2 - (|T_0\rangle_1 - |S\rangle_1)|\uparrow\downarrow\rangle_{a_1}|\uparrow\downarrow\rangle_2] \tag{7}$$

In the case of $P_{s,1} = 1$ and $P_{s,2} = 1$,

$$|\phi\rangle^d_{12a_1} = \frac{1}{2}[(|S\rangle_1 - |T_0\rangle_1)|\uparrow\downarrow\rangle_{a_1}|\downarrow\uparrow\rangle_2 + (|S\rangle_1 + |T_0\rangle_1)|\downarrow\uparrow\rangle_{a_1}|\uparrow\downarrow\rangle_2] \tag{8}$$

Then we measure the additional DQDM 1 under the basis $\{|S\rangle, |T_0\rangle\}$ and perform the single-qubit rotations σ_c and σ_t on DQDM 1 and DQDM 2, respectively, as that in Table 1. Thus the total state of DQDM 1 and DQDM 2 becomes

$$|\phi\rangle_{12} = \frac{1}{\sqrt{2}}(|S\rangle_1|T_0\rangle_2 + |T_0\rangle_1|S\rangle_2) \tag{9}$$

Table 1 Single-qubit rotations (σ_t and σ_c) corresponding to $P_{s,1}$, $P_{s,2}$ and the measurement result M_{a1} of additional DQDM 1

$P_{s,1}$	$P_{s,2}$	M_{a1}	σ_t	σ_c
0	0	$\vert S\rangle_{a1}$	I	I
0	0	$\vert T_0\rangle_{a1}$	σ_x	I
0	1	$\vert S\rangle_{a1}$	I	σ_z
0	1	$\vert T_0\rangle_{a1}$	σ_x	σ_z
1	0	$\vert S\rangle_{a1}$	I	I
1	0	$\vert T_0\rangle_{a1}$	σ_x	I
1	1	$\vert S\rangle_{a1}$	σ_x	σ_z
1	1	$\vert T_0\rangle_{a1}$	I	σ_z

This is a standard EPR state. Finally, we perform a single-qubit rotation σ_z on DQDM 1 and a Hadamard operation on DQDM 2, which lead the total state of DQDMs to

$$|\phi\rangle_{1,2,\ldots,N} = \frac{1}{\sqrt{2^N}}(|S\rangle_1\sigma_z^2 + |T_0\rangle_1)\bigotimes_{i=2}^{N}(|S\rangle_i + |T_0\rangle_i) \tag{10}$$

where $\sigma_z^2|T_0\rangle_2 = -|T_0\rangle_2$, $\sigma_z^2|S\rangle_2 = |S\rangle_2$. Then we perform above operations on DQDMs 2, 3 and additional DQDM 2 as on DQDMs 1, 2 and additional DQDM 1, which is shown in Fig. 1. The total state becomes

$$|\phi\rangle_{1,2,\ldots,N} = \frac{1}{\sqrt{2^N}}(|S\rangle_1\sigma_z^2 + |T_0\rangle_1)\otimes(|S\rangle_2\sigma_z^3 + |T_0\rangle_2)\bigotimes_{i=3}^{N}(|S\rangle_i + |T_0\rangle_i) \tag{11}$$

In succession, we perform these operations on DQDMs 3, 4 and additional DQDM 3, on DQDMs 4, 5 and additional DQDM 4, ..., finally, on DQDMs $N-1$, N and additional DQDM $N-1$, thus we obtain a multipartite cluster state as following

$$|\phi\rangle_{1,2,\ldots,N} = \frac{1}{\sqrt{2^N}}\bigotimes_{i=1}^{N}(|S\rangle_i\sigma_z^{i+1} + |T_0\rangle_i) \tag{12}$$

where $\sigma_z^{N+1} \equiv 1$.

3 Quantum State Transfer

Quantum state transfer is an important task for quantum communication. In order to transfer a state from one side (Alice) to another (Bob), suppose that there are four DQDMs A, B, C and D are in the following states

$$|\phi\rangle_{AB} = \frac{1}{\sqrt{2}}(|SS\rangle_{AB} + |T_0T_0\rangle_{AB}) \tag{13}$$

$$|\phi\rangle_C = a|T_0\rangle_C + b|S\rangle_C \tag{14}$$

$$|\phi\rangle_D = |T_0\rangle_D \tag{15}$$

where $|a|^2 + |b|^2 = 1$, DQDMs A, C and D belong to Alice and DQDM B belongs to Bob,which is shown in Fig. 2. The state of DQDM C want to be sent from Alice to

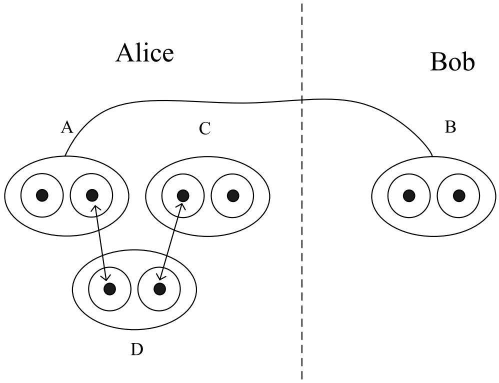

Fig. 2 The geometric arrangement of DQDMs

Bob. Firstly, Alice performs a Hadamard operation on DQDM A, i.e., $|T_0\rangle_A \rightarrow |\uparrow\downarrow\rangle_A$, $|S\rangle_A \rightarrow |\downarrow\uparrow\rangle_A$. Then she performs a spin-parity measurement on DQDMs A and D (the right electron of DQDM A and the left electron of DQDM D are used as "witness" states). If the result is even ($P_{s,A} = 1$), the state of total DQDMs will be

$$|\varphi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle_A|\downarrow\uparrow\rangle_D|T_0\rangle_B + |\downarrow\uparrow\rangle_A|\uparrow\downarrow\rangle_D|S\rangle_B)(a|T_0\rangle_C + b|S\rangle_C) \tag{16}$$

If the result is odd ($P_{s,A} = 0$), the state will be

$$|\varphi\rangle_{ABCD} = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle_A|\downarrow\uparrow\rangle_D|S\rangle_B + |\uparrow\downarrow\rangle_A|\uparrow\downarrow\rangle_D|T_0\rangle_B)(a|T_0\rangle_C + b|S\rangle_C) \tag{17}$$

Then she performs a Hadamard operation on DQDMs A, C and D, respectively, (16) and (17) become

$$|\varphi\rangle'_{ABCD} = \frac{1}{\sqrt{2}}(|T_0\rangle_A|S\rangle_D|T_0\rangle_B + |S\rangle_A|T_0\rangle_D|S\rangle_B)(a|\uparrow\downarrow\rangle_C + b|\downarrow\uparrow\rangle_C) \tag{18a}$$

$$|\varphi\rangle'_{ABCD} = \frac{1}{\sqrt{2}}(|S\rangle_A|S\rangle_D|S\rangle_B + |T_0\rangle_A|T_0\rangle_D|T_0\rangle_B)(a|\uparrow\downarrow\rangle_C + b|\downarrow\uparrow\rangle_C) \tag{18b}$$

Then she performs a spin-parity measurement on DQDMs D and C (the right electron of DQDM D and the left electron of DQDM C are used as "witness" states). Thus, in the case of $P_{s,A} = 1$, $P_{s,C} = 1$, the total state will be

$$\begin{aligned} |\psi\rangle^{(1)}_{ABCD} = \frac{1}{2}[&(a|T_0T_0\rangle_{AB} - b|T_0T_0\rangle_{AB} + a|SS\rangle_{AB} - b|SS\rangle_{AB})|\uparrow\downarrow\rangle_C|\downarrow\uparrow\rangle_D \\ &+ (-a|T_0T_0\rangle_{AB} - b|T_0T_0\rangle_{AB} + a|SS\rangle_{AB} + b|SS\rangle_{AB})|\downarrow\uparrow\rangle_C|\uparrow\downarrow\rangle_D] \end{aligned} \tag{19}$$

In the case of $P_{s,A} = 1$, $P_{s,C} = 0$, the total state will be

$$\begin{aligned} |\psi\rangle^{(2)}_{ABCD} = \frac{1}{2}[&(a|T_0T_0\rangle_{AB} + b|T_0T_0\rangle_{AB} + a|SS\rangle_{AB} + b|SS\rangle_{AB})|\downarrow\uparrow\rangle_C|\downarrow\uparrow\rangle_D \\ &+ (-a|T_0T_0\rangle_{AB} + b|T_0T_0\rangle_{AB} + a|SS\rangle_{AB} - b|SS\rangle_{AB})|\uparrow\downarrow\rangle_C|\uparrow\downarrow\rangle_D] \end{aligned} \tag{20}$$

In the case of $P_{s,A}=0$, $P_{s,C}=1$, the total state will be

$$|\psi\rangle^{(3)}_{ABCD}=\frac{1}{2}[(a|SS\rangle_{AB}-b|SS\rangle_{AB}+a|T_0T_0\rangle_{AB}-b|T_0T_0\rangle_{AB})|\uparrow\downarrow\rangle_C|\downarrow\uparrow\rangle_D + (-a|SS\rangle_{AB}-b|SS\rangle_{AB}+a|T_0T_0\rangle_{AB}+b|T_0T_0\rangle_{AB})|\downarrow\uparrow\rangle_C|\uparrow\downarrow\rangle_D] \quad (21)$$

In the case of $P_{s,A}=0$, $P_{s,C}=0$, the total state will be

$$|\psi\rangle^{(4)}_{ABCD}=\frac{1}{2}[(a|SS\rangle_{AB}+b|SS\rangle_{AB}+a|T_0T_0\rangle_{AB}+b|T_0T_0\rangle_{AB})|\downarrow\uparrow\rangle_C|\downarrow\uparrow\rangle_D + (-a|SS\rangle_{AB}+b|SS\rangle_{AB}+a|T_0T_0\rangle_{AB}-b|T_0T_0\rangle_{AB})|\uparrow\downarrow\rangle_C|\uparrow\downarrow\rangle_D] \quad (22)$$

Then Alice measures DQDM D under the basis $\{|S\rangle, |T_0\rangle\}$ and performs the single-qubit rotations σ'_c and σ'_t on DQDMs A and C, respectively, as that in Table 1. Thus the state of DQDMs A, B and C becomes

$$|\phi\rangle_{ABC}=\frac{1}{\sqrt{2}}(a|T_0\rangle_A|T_0\rangle_B|S\rangle_C+b|T_0\rangle_A|T_0\rangle_B|T_0\rangle_C+a|S\rangle_A|S\rangle_B|T_0\rangle_C+b|S\rangle_A|S\rangle_B|S\rangle_C) \quad (23)$$

Finally, Alice performs a Hadamard operation on DQDM A and measures DQDM A and C. If the result is $|T_0T_0\rangle_{AC}$, the state of DQDM B will be $|\Phi\rangle_B=a|S\rangle_B+b|T_0\rangle_B$; If the result is $|ST_0\rangle_{AC}$, the state of DQDM B will be $|\Phi\rangle_B=a|S\rangle_B-b|T_0\rangle_B$; If the result is $|T_0S\rangle_{AC}$, the state of DQDM B will be $|\Phi\rangle_B=a|T_0\rangle_B+b|S\rangle_B$; If the result is $|SS\rangle_{AC}$, the state of DQDM B will be $|\Phi\rangle_B=a|T_0\rangle_B-b|S\rangle_B$. When Alice tell her results to Bob, he can perform appropriate single-qubit operation on DQDM B, let

$$|\Phi\rangle'_B=a|T_0\rangle_B+b|S\rangle_B \quad (24)$$

This state is the same as the initial state of DQDM C, i.e., Alice's state has been transferred to Bob.

4 Discussions and Conclusions

In this section, we will discuss the feasibility of our schemes. In our double-quantum-dot molecule system, there is only one electron occupies in every quantum dot and tunneling coupling will happen between the two electrons. By adjusting the gate-bias voltage, the charge state can be changed between (0, 2) and (1, 1) [22, 23]. We choose the spin singlet state $|S\rangle$ and triplet state $|T_0\rangle$ as qubit since they are both the lowest eigenstates of the system. Generally, there are two important sources for electron spin in quantum-dot system, one is that the spin-orbit interaction couples electron spins to their orbital state, where spins will be indirectly sensitive to fluctuations in electron environment, the other is the Fermi contact hyperfine interaction between electrons and surrounding nuclear spins where spin will rapidly decay if fluctuations in the nuclear spin environment are not properly controlled. It is shown that the effect of spin-orbit coupling can be strongly suppressed for the confined electrons in weak magnetic field [26, 27].

Quantum measurement is another important question in QIP. The single-qubit rotations of the qubit could be performed by using an inhomogeneous magnetic field. Electron-spin parity directly measurement is still very difficult, but one can convert spin parity measurement into charge measurement, which can be realized in experiment [28, 29]. The spin-to-charge conversion has been extensively researched both theoretically and experimentally.

The freedom of electron and the freedom of spin are incoherent, so the electron measurement will not destroy the electronic spin. That is to say, the spin parity measurement is nondestructive measurement, which is very important for realizing scalable quantum computation in electronic nanostructures. The single-qubit measurement in current scheme also can be implemented by spin-to-charge conversion or quantum-point-contact measurements [30].

In conclusion, we propose two physical schemes via double-quantum-dot molecule system: one is entanglement generation, the other is entanglement state transfer. The main quantum measurement are electron-spin parity measurement and single-qubit measurement rather than the routine Bell state measurement. The measurements are both nondestructive, which is very important for quantum communication, specially for quantum feedback control.

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